NONSTATIONARY HEAT CONDUCTIVITY OF MOIST BODIES

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The temperature field of moist bodies in the form of a plate, a hollow cylinder, and a hollow sphere is analyzed for general boundary conditions of the third kind at the inner and outer surfaces.

The heating of bodies of various configurations for boundary conditions of the third kind has been examined in [1]. In the case of additional heat input by radiation (convective–radiative desiccation) at the boundaries of hollow bodies, it is advantageous to use general boundary conditions of the third kind (which differ at the inner and outer surface). For a plate, we shall examine also asymmetric heating, in which case the plate coordinates will be chosen similarly to those of hollow bodies. This corresponds to the case of a plane inner channel with a width of $2r_0$ and a wall thickness of $R - r_0$. In the general case, the evaporation rate and the desiccation factor differ at the inner and outer surfaces of bodies.

It is assumed [1] that the evaporation rate is constant during the initial phase of desiccation and that it varies with time according to a power law in the main phase. The value of q_{re} is constant, since in the case where the temperatures of the radiator and moist body differ widely it is the radiator temperature that determines the value of q_{re}. The problem is formulated as follows:

$$\frac{\partial \theta}{\partial \mathrm{Fo}} = \frac{\partial^2 \theta}{\partial r^2} + \frac{\beta}{r}\frac{\partial \theta}{\partial r}, \tag{1}$$

$$\frac{r_0}{R} < r < 1; \quad \mathrm{Fo} > 0;$$

$$\theta(r,\ 0) = 0; \tag{2}$$

$$-\frac{\partial \theta\left(\frac{r_0}{R},\ \mathrm{Fo}\right)}{\partial r} = \mathrm{Ki}_{\mathrm{T}_0} - \mathrm{Po}_0 \exp(-\mathrm{Pd}_0\mathrm{Fo}) + \mathrm{Bi}_0\left[\theta_{c_0} - \theta\left(\frac{r_0}{R},\ \mathrm{Fo}\right)\right]; \tag{3}$$

$$\frac{\partial \theta(1,\ \mathrm{Fo})}{\partial r} = \mathrm{Ki}_{\mathrm{T}_1} - \mathrm{Po}_1 \exp(-\mathrm{Pd}_1\mathrm{Fo}) + \mathrm{Bi}_1[\theta_{c_1} - \theta(1,\ \mathrm{Fo})]. \tag{4}$$

For a plate, we have $\beta = 0$; for a cylinder $\beta = 1$, and for a sphere $\beta = 2$. The Pomerantsev number, Po, characterizes the intensity of internal negative heat sources; in the case of desiccation, $\mathrm{Po} = \mathrm{Ki}_M\mathrm{LuKo}$.

We use Laplace transforms with respect to the Fourier variable to solve the problem (1)-(4).

The final solutions for the transforms are as follows:

for a plate

$$\theta(r,\ \mathrm{Fo}) = p - 2\left(1 - \frac{r_0}{R}\right)\sum_{n=1}^{\infty}\left\{\left[M_0\left(1 - \frac{r_0}{R}\right)^2 - N_0\mu_n^2\right]S_1(\mu_n)/\varphi_0(\mu_n)\right.$$

$$\left.+\left[M_1\left(1 - \frac{r_0}{R}\right)^2 - N_1\mu_n^2\right]S_2(\mu_n)/\varphi_1(\mu_n)\right\}\exp\left[-\mu_n^2\mathrm{Fo}\Big/\left(1 - \frac{r_0}{R}\right)^2\right]\Big/W(\mu_n), \tag{5}$$

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$$S_1(\mu_n) = \mathrm{Bi}_1\left(1-\frac{r_0}{R}\right)\sin[\mu_n R(1-r)/(R-r_0)]+\mu_n\cos[\mu_n R(1-r)/(R-r_0)],$$

$$S_2(\mu_n) = \mathrm{Bi}_0\left(1-\frac{r_0}{R}\right)\sin[\mu_n (Rr-r_0)/(R-r_0)]+\mu_n\cos[\mu_n (Rr-r_0)/(R-r_0)],$$

$$W(\mu_n)=\left[(\mathrm{Bi}_0+\mathrm{Bi}_1)\left(1-\frac{r_0}{R}\right)+2\right]\mu_n^2\sin\mu_n-\left[\mathrm{Bi}_0\mathrm{Bi}_1\left(1-\frac{r_0}{R}\right)^2-\mu_n^2\right](\cos\mu_n-\sin\mu_n),$$

$$p=\frac{(\mathrm{Ki}_{T_0}+\mathrm{Bi}_0\,\theta_{c_0})[\mathrm{Bi}_1(1-r)+1]+(\mathrm{Ki}_{T_1}+\mathrm{Bi}_1\,\theta_{c_1})\left[\mathrm{Bi}_0\left(r-\frac{r_0}{R}\right)+1\right]}{\mathrm{Bi}_0+\mathrm{Bi}_1+\mathrm{Bi}_0\mathrm{Bi}_1\left(1-\frac{r_0}{R}\right)}$$
$$-\frac{\mathrm{Po}_0[\mathrm{Bi}_1\sin\sqrt{\mathrm{Pd}_0}(1-r)+\sqrt{\mathrm{Pd}_0}\cos\sqrt{\mathrm{Pd}_0}(1-r)]\exp(-\mathrm{Pd}_0\mathrm{Fo})}{(\mathrm{Bi}_0+\mathrm{Bi}_1)\sqrt{\mathrm{Pd}_0}\cos\sqrt{\mathrm{Pd}_0}\left(1-\frac{r_0}{R}\right)+(\mathrm{Bi}_1\mathrm{Bi}_0-\mathrm{Pd}_0)\sin\sqrt{\mathrm{Pd}_0}\left(1-\frac{r_0}{R}\right)}$$
$$-\frac{\mathrm{Po}_1\left[\mathrm{Bi}_0\sin\sqrt{\mathrm{Pd}_1}\left(r-\frac{r_0}{R}\right)+\sqrt{\mathrm{Pd}_1}\cos\sqrt{\mathrm{Pd}_1}\left(r-\frac{r_0}{R}\right)\right]\exp(-\mathrm{Pd}_1\mathrm{Fo})}{(\mathrm{Bi}_0+\mathrm{Bi}_1)\sqrt{\mathrm{Pd}_1}\cos\sqrt{\mathrm{Pd}_1}\left(1-\frac{r_0}{R}\right)+(\mathrm{Bi}_1\mathrm{Bi}_0-\mathrm{Pd}_1)\sin\sqrt{\mathrm{Pd}_1}\left(1-\frac{r_0}{R}\right)}, \tag{6}$$

where μ_n are the roots of the characteristic equation

$$\mathrm{tg}\,\mu=-\frac{\mu(\mathrm{Bi}_0+\mathrm{Bi}_1)\left(1-\frac{r_0}{R}\right)}{\mathrm{Bi}_0\mathrm{Bi}_1\left(1-\frac{r_0}{R}\right)^2-\mu^2}; \tag{7}$$

for a cylinder

$$\theta(r,\ \mathrm{Fo})=p-\sum_{n=1}^{\infty}\frac{2(M_0-N_0\mu_n^2)[\mathrm{Bi}_1V_{00}(\mu_n r)+\mu_n V_{10}(\mu_n r)]\exp(-\mu_n^2\mathrm{Fo})}{\mu_n(\mathrm{Pd}_0-\mu_n^2)\psi(\mu_n)}$$
$$-\sum_{n=1}^{\infty}\frac{2(M_1-N_1\mu_n^2)[\mathrm{Bi}_0F_{00}(\mu_n r)+\mu_n F_{01}(\mu_n r)]\exp(-\mu_n^2\mathrm{Fo})}{\mu_n(\mathrm{Pd}_1-\mu_n^2)\psi(\mu_n)}, \tag{8}$$

$$p=\frac{(\mathrm{Ki}_{T_0}+\mathrm{Bi}_0\,\theta_{c_0})(\mathrm{Bi}_1\ln r-1)-(\mathrm{Ki}_{T_1}+\mathrm{Bi}_1\,\theta_{c_1})\left(\mathrm{Bi}_0\ln\frac{rR}{r_0}+\frac{R}{r_0}\right)}{\mathrm{Bi}_0\mathrm{Bi}_1\ln\frac{r_0}{R}-\mathrm{Bi}_1\frac{R}{r_0}-\mathrm{Bi}_0}$$
$$-\frac{\mathrm{Po}_0[\mathrm{Bi}_1V_{00}(\sqrt{\mathrm{Pd}_0}r)-\sqrt{\mathrm{Pd}_0}V_{10}(\sqrt{\mathrm{Pd}_0}r)]\exp(-\mathrm{Pd}_0\mathrm{Fo})}{\mathrm{Bi}_1\mathrm{Bi}_0V_{00}(\sqrt{\mathrm{Pd}_0})+\sqrt{\mathrm{Pd}_0}[\mathrm{Bi}_1V_{01}(\sqrt{\mathrm{Pd}_0})-\mathrm{Bi}_0V_{10}(\sqrt{\mathrm{Pd}_0})-\sqrt{\mathrm{Pd}_0}V_{11}(\sqrt{\mathrm{Pd}_0})]}$$
$$-\frac{\mathrm{Po}_1[\mathrm{Bi}_0V_{00}(\sqrt{\mathrm{Pd}_1}r)+\sqrt{\mathrm{Pd}_1}V_{01}(\sqrt{\mathrm{Pd}_1}r)]\exp(-\mathrm{Pd}_1\mathrm{Fo})}{\mathrm{Bi}_1\mathrm{Bi}_0V_{00}(\sqrt{\mathrm{Pd}_1})+\sqrt{\mathrm{Pd}_1}[\mathrm{Bi}_1V_{01}(\sqrt{\mathrm{Pd}_1})-\mathrm{Bi}_0V_{10}(\sqrt{\mathrm{Pd}_1})-\sqrt{\mathrm{Pd}_1}V_{11}\sqrt{\mathrm{Pd}_1})]}, \tag{9}$$

$$\psi(\mu_n)=\mathrm{Bi}_1\mathrm{Bi}_0\left[\frac{r_0}{R}V_{10}(\mu_n)+V_{01}(\mu_n)\right]-\mathrm{Bi}_1\mu_n\left[\frac{r_0}{R}V_{00}(\mu_n)-V_{11}(\mu_n)\right]$$
$$-\mathrm{Bi}_0\mu_n\left[\frac{r_0}{R}V_{11}(\mu_n)-V_{00}(\mu_n)\right]+\mu_n^2\left[\frac{r_0}{R}V_{01}(\mu_n)+V_{10}(\mu_n)\right], \tag{10}$$

$$V_{kj}(\sqrt{\mathrm{Pd}_0}r)=J_k(\sqrt{\mathrm{Pd}_0})Y_j(\sqrt{\mathrm{Pd}_0}r)-Y_k(\sqrt{\mathrm{Pd}_0})J_j(\sqrt{\mathrm{Pd}_0}r), \tag{11}$$

$$V_{kj}(\sqrt{\mathrm{Pd}_1}r)=J_k(\sqrt{\mathrm{Pd}_1}r)Y_j\left(\sqrt{\mathrm{Pd}_1}\frac{r_0}{R}\right)-Y_k(\sqrt{\mathrm{Pd}_1}r)J_j\left(\sqrt{\mathrm{Pd}_1}\frac{r_0}{R}\right), \tag{12}$$

$$V_{kj}(\sqrt{\mathrm{Pd}_0})=J_k(\sqrt{\mathrm{Pd}_0})Y_j\left(\sqrt{\mathrm{Pd}_0}\frac{r_0}{R}\right)-Y_k(\sqrt{\mathrm{Pd}_0})J_j\left(\sqrt{\mathrm{Pd}_0}\frac{r_0}{R}\right), \tag{13}$$

$$V_{kj}(\sqrt{\mathrm{Pd}_1})=J_k(\sqrt{\mathrm{Pd}_1})Y_j\left(\sqrt{\mathrm{Pd}_1}\frac{r_0}{R}\right)-Y_k(\sqrt{\mathrm{Pd}_1})J_j\left(\sqrt{\mathrm{Pd}_1}\frac{r_0}{R}\right), \tag{14}$$

$$V_{kj}(\mu_n)=J_k(\mu_n)Y_j\left(\mu_n\frac{r_0}{R}\right)-Y_k(\mu_n)J_j\left(\mu_n\frac{r_0}{R}\right), \tag{15}$$

$$V_{kj}(\mu_n r) = J_k(\mu_n) Y_j(\mu_n r) - Y_k(\mu_n) J_j(\mu_n r), \tag{16}$$

$$F_{kj}(\mu_n r) = J_k(\mu_n r) Y_j\left(\mu_n \frac{r_0}{R}\right) - Y_k(\mu_n r) J_j\left(\mu_n \frac{r_0}{R}\right), \tag{17}$$

$$k = 0;\ 1; \quad j = 0;\ 1,$$

where μ_n are the roots of the characteristic equation

$$\mathrm{Bi}_1\mathrm{Bi}_0 V_{00}(\mu) + \mathrm{Bi}_1 \mu V_{01}(\mu) - \mathrm{Bi}_0 \mu V_{10}(\mu) - \mu^2 V_{11}(\mu) = 0; \tag{18}$$

for a sphere

$$\theta(r,\ \mathrm{Fo}) = p - 2\left(1 - \frac{r_0}{R}\right)\sum_{n=1}^{\infty}\left\{ r_0\left[M_0\left(1-\frac{r_0}{R}\right)^2 - N_0\mu_n^2\right] T_1(\mu_n)/rR\varphi_0(\mu_n)\right.$$

$$\left. + \left[M_1\left(1-\frac{r_0}{R}\right)^2 - N_1\mu_n^2\right] T_2(\mu_n)/r\,\varphi_1(\mu_n)\right\}\sin\mu_n \exp\left[-\mu_n^2\mathrm{Fo}\Big/\left(1-\frac{r_0}{R}\right)^2\right]\Big/ G(\mu_n), \tag{19}$$

$$T_1(\mu_n) = (\mathrm{Bi}_1 - 1)\left(1 - \frac{r_0}{R}\right)\sin[\mu_n R(1-r)/(R-r_0)] + \mu_n\cos[\mu_n R(1-r)/(R-r_0)],$$

$$T_2(\mu_n) = \left(\frac{R}{r_0} + \mathrm{Bi}_0\right)\left(1 - \frac{r_0}{R}\right)\sin[\mu_n(Rr - r_0)/(R-r_0)] + \mu_n\cos[\mu_n(Rr-r_0)/(R-r_0)],$$

$$G(\mu_n) = \left[\left(\frac{R}{r_0} + \mathrm{Bi}_0\right)(\mathrm{Bi}_1 - 1)\left(1-\frac{r_0}{R}\right)^2 + \mu_n^2\right]\sin^2\mu_n + \left(\mathrm{Bi}_0 + \mathrm{Bi}_1 + \frac{R}{r_0} - 1\right)\left(1 - \frac{r_0}{R}\right)\mu_n^2,$$

$$p = \frac{r_0(\mathrm{Ki}_{T_0} + \mathrm{Bi}_0\theta_{c_0})[(\mathrm{Bi}_1 - 1)(1-r)+1]}{rR\left(\mathrm{Bi}_1\frac{R}{r_0} + \mathrm{Bi}_1\mathrm{Bi}_0 + \mathrm{Bi}_0\frac{r_0}{R} - \mathrm{Bi}_1\mathrm{Bi}_0\frac{r}{R}\right)} + \frac{R(\mathrm{Ki}_{T_1} + \mathrm{Bi}_1\theta_{c_1})\left[\left(\frac{R}{r_0} + \mathrm{Bi}_0\right)\left(r - \frac{r_0}{R}\right) + 1\right]}{rR\left(\mathrm{Bi}_1\frac{R}{r_0} + \mathrm{Bi}_1\mathrm{Bi}_0 + \mathrm{Bi}_0\frac{r_0}{R} - \mathrm{Bi}_1\mathrm{Bi}_0\frac{r_0}{R}\right)}$$

$$- \frac{r_0\mathrm{Po}_0\left\{(\mathrm{Bi}_1 - 1)\sin\left[\sqrt{\mathrm{Pd}_0}(1-r)\right] + \sqrt{\mathrm{Pd}_0}\cos\left[\sqrt{\mathrm{Pd}_0}(1-r)\right]\right\}}{rR\left\{(L_0 - \mathrm{Pd}_0)\sin\left[\sqrt{\mathrm{Pd}_0}\left(1-\frac{r_0}{R}\right)\right] + L_1\sqrt{\mathrm{Pd}_0}\cos\left[\sqrt{\mathrm{Pd}_0}\left(1-\frac{r_0}{R}\right)\right]\right\}}\exp(-\mathrm{Pd}_0\mathrm{Fo})$$

$$- \frac{\mathrm{Po}_1\left\{\left(\frac{R}{r_0} + \mathrm{Bi}_0\right)\sin\left[\sqrt{\mathrm{Pd}_1}\left(r - \frac{r_0}{R}\right)\right] + \sqrt{\mathrm{Pd}_1}\cos\left[\sqrt{\mathrm{Pd}_1}\left(r - \frac{r_0}{R}\right)\right]\right\}}{r\left\{(L_0 - \mathrm{Pd}_1)\sin\left[\sqrt{\mathrm{Pd}_1}\left(1-\frac{r_0}{R}\right)\right] + L_1\sqrt{\mathrm{Pd}_1}\cos\left[\sqrt{\mathrm{Pd}_1}\left(1-\frac{r_0}{R}\right)\right]\right\}}\exp(-\mathrm{Pd}_1\mathrm{Fo}), \tag{20}$$

where μ_n are the roots of the characteristic equation

$$\mathrm{tg}\,\mu = -\frac{\mu\left(\mathrm{Bi}_0 + \mathrm{Bi}_1 + \frac{R}{r_0} - 1\right)\left(1 - \frac{r_0}{R}\right)}{\left(\frac{R}{r_0} + \mathrm{Bi}_0\right)(\mathrm{Bi}_1 - 1)\left(1 - \frac{r_0}{R}\right)^2 - \mu^2}. \tag{21}$$

The values of the first roots of Eqs. (7), (18), and (21) for $\mathrm{Bi}_0 = \mathrm{Bi}_1 = \mathrm{Bi}$ are compiled in Table 1.

If boundary conditions of the second kind ($\mathrm{Bi}_0 = \mathrm{Bi}_1 = 0$) are given at the inner and outer surfaces of the bodies, the quantity p has the following values:

for a plate

$$p = \frac{(\mathrm{Ki}_{T_0} + \mathrm{Ki}_{T_1})}{1 - \frac{r_0}{R}}\left[\mathrm{Fo} - \frac{\left(1 - \frac{r_0}{R}\right)^3}{6}\right] + \frac{\mathrm{Ki}_{T_0}(1-r)^2}{2\left(1 - \frac{r_0}{R}\right)} + \frac{\mathrm{Ki}_{T_1}\left(r - \frac{r_0}{R}\right)^2}{2\left(1 - \frac{r_0}{R}\right)}$$

$$- \frac{\mathrm{Po}_0}{\mathrm{Pd}_0\left(1 - \frac{r_0}{R}\right)} - \frac{\mathrm{Po}_1}{\mathrm{Pd}_1\left(1 - \frac{r_0}{R}\right)} + \frac{\mathrm{Po}_0\cos\sqrt{\mathrm{Pd}_0}(1-r)\exp(-\mathrm{Pd}_0\mathrm{Fo})}{\sqrt{\mathrm{Pd}_0}\sin\sqrt{\mathrm{Pd}_0}\left(1 - \frac{r_0}{R}\right)}$$

$$+ \frac{\mathrm{Po}_1\cos\sqrt{\mathrm{Pd}_1}(1-r)\exp(-\mathrm{Pd}_1\mathrm{Fo})}{\sqrt{\mathrm{Pd}_1}\sin\sqrt{\mathrm{Pd}_1}\left(1 - \frac{r_0}{R}\right)}; \tag{22}$$

TABLE 1. Roots of the Characteristic Equations, μ

Bi	Roots of equation (7) (plate)				Roots of equation (18) (cylinder)				Roots of equation (21) (sphere)			
	r_0/R											
	0,2	0,4	0,6	0,8	0,2	0,4	0,6	0,8	0,2	0,4	0,6	0,8
0	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
0,02	0,1787	0,1548	0,1264	0,0932	0,1796	0,2532	0,3142	0,4442	0,1999	0,1636	0,1289	0,0898
0,04	0,2523	0,2187	0,1786	0,1264	0,2409	0,3613	0,4438	0,6302	0,2830	0,2311	0,1822	0,1269
0,06	0,3086	0,2675	0,2187	0,1548	0,2995	0,4432	0,5442	0,7723	0,3459	0,2825	0,2231	0,1555
0,08	0,3559	0,3086	0,2523	0,1786	0,3411	0,5131	0,6290	0,8934	0,3984	0,3255	0,2573	0,1793
0,1	0,3974	0,3447	0,2819	0,1997	0,3816	0,5724	0,7043	0,9984	0,4448	0,3637	0,2876	0,2005
0,2	0,5583	0,4851	0,3973	0,2819	0,5271	0,8448	0,9927	1,4936	0,6238	0,5113	0,4052	0,2831
0,4	0,7993	0,6792	0,5583	0,3979	0,9837	1,1291	1,3951	1,9863	0,8672	0,7145	0,5688	0,3990
0,6	0,9423	0,8239	0,6792	0,4851	1,1694	1,3700	1,6971	2,4247	1,0445	0,8649	0,6918	0,4870
0,8	1,0745	0,9423	0,7729	0,5583	1,3309	1,5658	1,9467	2,7911	1,1865	0,9873	0,7932	0,5605
1,0	1,1865	1,0436	0,8657	0,6221	1,4670	1,7334	2,1622	3,1103	1,3053	1,0915	0,8807	0,6245
2,0	1,5824	1,4102	1,1865	0,8657	1,9422	2,3373	2,9621	4,3276	1,7115	1,4633	1,2041	0,8687
4,0	2,0169	1,8357	1,5828	1,1865	2,4552	3,0342	3,9467	5,9310	2,1286	1,8836	1,5998	1,1900
6,0	2,2612	2,0898	1,8350	1,4102	2,7417	3,4487	4,5766	7,0421	2,3496	2,1290	1,8516	1,4137
8,0	2,4189	2,2612	2,0169	1,5830	2,9278	3,7165	5,0268	7,9071	2,4886	2,2929	2,0307	1,5855
10,0	2,5292	2,3850	2,1538	1,7207	3,0579	3,8819	5,3669	8,5987	2,5848	2,4108	2,1656	1,7240
20,0	2,7956	2,6992	2,5292	2,1538	3,3795	4,4466	6,2998	10,7621	2,8183	2,7101	2,5351	2,1561
40,0	2,9574	2,9011	2,7956	2,5292	3,5801	4,7805	6,9626	12,6373	2,9647	2,9047	2,7979	2,5304
60,0	3,0162	2,9767	2,9011	2,6992	3,6544	4,9060	7,2261	13,4850	3,0198	2,9784	2,9022	2,6998
80,0	3,0465	3,0162	2,9575	2,7956	3,6930	4,9725	7,3672	13,9684	3,0486	3,0172	2,9580	2,7961
100,0	3,0651	3,0404	2,9923	2,8578	3,7167	5,0132	7,4547	14,2782	3,0665	3,0411	2,9927	2,8580
∞	3,1416	3,1416	3,1416	3,1416	3,8211	5,1834	7,8312	15,6820	3,1416	3,1416	3,1416	3,1416

for a cylinder

$$
\begin{aligned}
p = {} & \frac{\mathrm{Ki}_{T_0}}{\left(\frac{R}{r_0} - \frac{r_0}{R}\right)}\left[2\mathrm{Fo} + \frac{r^2-1}{2} - \frac{\frac{R}{r_0} - \frac{r_0^3}{R^3}}{4\left(\frac{R}{r_0} - \frac{r_0}{R}\right)}\right] \\
& + \frac{\mathrm{Ki}_{T_1}}{\frac{R}{r_0} - \frac{r_0}{R}}\left[2\mathrm{Fo}\frac{R}{r_0} + \frac{1}{2}\left(\frac{Rr^2}{r_0} - \frac{r_0}{R}\right) - \frac{1}{4}\left(\frac{R}{r_0} + \frac{r_0}{R}\right)\right] \\
& - \frac{\mathrm{Po}_0}{\mathrm{Pd}_0}\left[\frac{2}{\frac{R}{r_0} - \frac{r_0}{R}} + \frac{\sqrt{\mathrm{Pd}_0}\,V_{10}(\sqrt{\mathrm{Pd}_0}\,r)\exp(-\mathrm{Pd}_0\mathrm{Fo})}{V_{11}(\sqrt{\mathrm{Pd}_0})}\right] \\
& - \frac{\mathrm{Po}_1}{\mathrm{Pd}_1}\left[\frac{2r_0^2}{R^2 - r_0^2} - \frac{\sqrt{\mathrm{Pd}_1}\,V_{10}(\sqrt{\mathrm{Pd}_1}\,r)\exp(-\mathrm{Pd}_1\mathrm{Fo})}{V_{11}(\sqrt{\mathrm{Pd}_1})}\right];
\end{aligned}
\tag{23}
$$

for a sphere

$$
\begin{aligned}
p = {} & \frac{\mathrm{Ki}_{T_0}}{\frac{R}{r_0} - \frac{r_0^2}{R^2}}\left[3\mathrm{Fo} + \frac{1}{r} - \frac{3}{2} + \frac{r^2}{2} - \frac{\frac{3}{10}\left(\frac{R^2}{r_0^2} - \frac{r_0^3}{R^3}\right) + \frac{3}{2}\left(\frac{r_0}{R} - 1\right)}{\frac{R}{r_0} - \frac{r_0^2}{R^2}}\right] \\
& + \frac{\mathrm{Ki}_{T_1}}{\frac{R}{r_0} - \frac{r_0^2}{R^2}}\left[3\mathrm{Fo}\frac{R}{r_0} + \frac{r^2R}{2r_0} - \frac{3r_0}{2R} + \frac{r_0^2}{rR^2} - \frac{\frac{3}{10}\left(\frac{R^3}{r_0^3} - \frac{r_0^2}{R^2}\right) + \frac{3}{2}\left(1 - \frac{R}{r_0}\right)}{\frac{R}{r_0} - \frac{r_0^2}{R^2}}\right] \\
& - \frac{3\mathrm{Po}_0}{\mathrm{Pd}_0\left(\frac{R}{r_0} - \frac{r_0^2}{R^2}\right)} - \frac{3\mathrm{Po}_1 R}{\mathrm{Pd}_1 r_0\left(\frac{R}{r_0} - \frac{r_0^2}{R^2}\right)} \\
& + \frac{\mathrm{Po}_0\left[\sqrt{\mathrm{Pd}_0}\cos\sqrt{\mathrm{Pd}_0}(1-r) - \sin\sqrt{\mathrm{Pd}_0}(1-r)\right]\exp(-\mathrm{Pd}_0\mathrm{Fo})}{r\frac{R}{r_0}\left[\sqrt{\mathrm{Pd}_0}\left(\frac{R}{r_0} - 1\right)\cos\sqrt{\mathrm{Pd}_0}\left(1 - \frac{r_0}{R}\right) - \left(\mathrm{Pd}_0 - \frac{R}{r_0}\right)\sin\sqrt{\mathrm{Pd}_0}\left(1 - \frac{r_0}{R}\right)\right]} \\
& + \frac{\mathrm{Po}_1\left[\frac{R}{r_0}\sin\sqrt{\mathrm{Pd}_1}\left(r - \frac{r_0}{R}\right) + \sqrt{\mathrm{Pd}_1}\cos\sqrt{\mathrm{Pd}_1}\left(r - \frac{r_0}{R}\right)\right]\exp(-\mathrm{Pd}_1\mathrm{Fo})}{r\left[\sqrt{\mathrm{Pd}_1}\left(\frac{R}{r_0} - 1\right)\cos\sqrt{\mathrm{Pd}_1}\left(1 - \frac{r_0}{R}\right) - \left(\mathrm{Pd}_1 - \frac{R}{r_0}\right)\sin\sqrt{\mathrm{Pd}_1}\left(1 - \frac{r_0}{R}\right)\right]}.
\end{aligned}
\tag{24}
$$

In expressions (5), (8), (19):

$$\begin{aligned} M_0 &= \mathrm{Pd}_0 (\mathrm{Ki}_{T_0} + \mathrm{Bi}_0 \theta_{c_0}), \\ N_0 &= \mathrm{Ki}_{T_0} + \mathrm{Bi}_0 \theta_{c_0} - \mathrm{Po}_0, \\ M_1 &= \mathrm{Pd}_1 (\mathrm{Ki}_{T_1} + \mathrm{Bi}_1 \theta_{c_1}), \\ N_1 &= \mathrm{Ki}_{T_1} + \mathrm{Bi}_1 \theta_{c_1} - \mathrm{Po}_1. \end{aligned} \tag{25}$$

For desiccation during the initial phase, Pd = 0; for heating of dry bodies, Po = 0. For $r_0 = 0$ (correspondingly, setting all dimensionless numbers which refer to the inner surface of the body equal to zero) and $\mathrm{Ki}_{T1} = 0$, we obtain the solutions for solid moist bodies with boundary conditions of the third kind that are given in [1].

By assuming that the values of Bi tend to infinity, or that the corresponding values of Bi of Ki_T are zero, from the general equations (5), (8), (19), one can obtain solutions for various combinations of boundary conditions of the first, second, and third kind at the outer and inner surfaces of moist hollow bodies.

NOTATION

r_0, R, x	are the inner, outer, and instantaneous coordinates of a body, respectively;
t_0, t, and t_c	are the inner and instantaneous temperatures of a body and the ambient temperature, respectively;
$\theta = (t - t_0)/t_0$	is the dimensionless temperature of body;
$\theta_c = (t_c - t_0)/t_0$	is the dimensionless ambient temperature;
$r = x/R$	is the dimensionless coordinate of a point on the body;
m	is the maximum rate of moisture evaporation (referred to the unit area of the body);
ρ	is the heat of evaporation;
k	is the desiccation coefficient;
τ	is the time;
q_{re}	is the thermal radiation flux at the body's surface;
α	is the heat-transfer coefficient at the body's surface;
a, λ, a', u	are the thermal diffusivity, thermal conductivity, mass transfer, and initial moisture of the body, respectively;
c, γ	are the heat capacity and density of the body, respectively;
$\mathrm{Fo} = a\tau/R^2$	is the Fourier number;
$\mathrm{Ki}_T = q_{re}R/\lambda t_0$	is the Kirpichev's heat-transfer ratio;
$\mathrm{Ki}_M = mR/a'u\gamma$	is the Kirpichev's mass-transfer ratio;
$\mathrm{Lu} = a'/a$	is the Lykov's ratio;
$\mathrm{Ko} = \rho u/ct_0$	is the Kossovich's ratio;
$\mathrm{Po} = \rho mR/\lambda t_0$	is the Pomerantsev's ratio;
$\mathrm{Pd} = kR^2/a$	is the Predvoditelev's ratio;
J_{kj}, Y_{kj}	are the Bessel functions of the first and second kind;
$\mathrm{Bi} = \alpha R/\lambda$	is the Biot number.

Subscripts

0 denotes the inner surface of the body;

1 denotes the outer surface of the body;

$$\varphi_0(\mu_n) = \mathrm{Pd}_0\left(1 - \frac{r_0}{R}\right)^2 - \mu_n^2; \quad \varphi_1(\mu_n) = \mathrm{Pd}_1\left(1 - \frac{r_0}{R}\right)^2 - \mu_n^2;$$

$$L_0 = \left(\frac{R}{r_0} + \mathrm{Bi}_0\right)(\mathrm{Bi}_1 - 1); \quad L_1 = \mathrm{Bi}_0 + \mathrm{Bi}_1 + \frac{R}{r_0} - 1.$$

LITERATURE CITED

1. A. V. Lykov, Theory of Heat Conductivity [in Russian], Vysshaya Shkola (1967).